ANNOTATION

dissertation for a degree of Philosophy Doctor (PhD) in
the specialty «8D05306-Physics» Serikbolova Albina Askarovna on the topic
«**Branes and monopoles in modified gravities and Yang-Mills theories»**

General description of the research

The dissertation work presents the results of the research of the mass gap in the energy spectrum of a monopole-like object with nonlinear spinor source and hypothetical astrophysical objects in multidimensional space-time – thick branes ($\mathcal{D}$-branes) in the framework of the modified gravity.

Relevance of the topic

In modern physics for understanding and describing the structure and evolution of the Universe there is a necessity for the investigation of models of the Universe in higher-dimensional space-time. There are huge applications of multidimensional theory in a wide range of physics, such as string theory, GUT, cosmology and especially in high-energy physics for solving different problems such as the problem of mass hierarchy, stability of the proton, ect.

The first idea of considering higher-dimensional space-time belongs to Kaluza and Klein, who created a 5-dimensional theory, which is the unification of gravitational and electromagnetic interactions. Later, superstring theories requiring extra space dimensions were created.

Another argument in support of a transition to the geometry of a higher-dimensional space is the possibility to analyze and investigate various hypothetical objects: domain walls, thick branes. It is supposed that we live in a thin brane, which is n-dimensional hypersurfaces embedded in the multidimensional space-time (bulk). Within the framework of these models of the world on the brane, it is possible to naturally describe the hierarchy of masses of elementary particles, as well as to solve a number of other problems of the theory of elementary particles. $\mathcal{D}$–branes in string theory are hypothetical fundamental multidimensional physical objects on which open strings are associated with Dirichlet boundary conditions. These objects were first predicted within the framework of Einstein's theory of gravity.

The first part of the work consists of the considering regular solutions of the gravitational equations describing thick branes ($\mathcal{D}$– branes) in multidimensional space-time within the framework of $\mathcal{F}(R)$ modified theories. The first thing that needs to be said is that modified theories of gravity can be interpreted as an alternative to the cosmological constant (or dark energy) for explaining the accelerated expansion of the Universe. This is a completely new approach in describing the accelerated expansion of the Universe, since we do not use the old methods of GR and try to modify them. Therefore, these theories are called modified theories of gravity (MG). One of the most striking features of these theories is that

the Lagrangian density is no longer the scalar curvature R (as in GR), but some non-linear function of the scalar curvature $\mathcal{F}(R)$.

The second part of the work is consist of considering magnetic monopoles ($\mathcal{M}s$) within the framework of non-Abelian Yang-Mills fields interacting with a spinor field. A magnetic monopole is a hypothetical elementary particle with nonzero magnetic charge. There is no real physical confirmation of the existence of a magnetic monopole. In 1931 Paul Dirac suggested that from the asymmetric QED, a symmetric QED can be constructed by including a magnetic charge. The hypothetical magnetic charge which was proposed by Dirac is called Dirac monopole. Later, magnetic monopoles in non-Abelian gauge theories were discovered independently by Gerard 't Hooft and Alexander Polyakov.

90 years have passed, but the problem of the existence of a magnetic monopole is still **relevant**, and more and more experiments are being carried out to solve it. It is important to investigate properties of a magnetic monopole like: magnetic field strength, energy spectrum. They find their applications in a huge variety of topics in theoretical physics, including problems in the standard model, GUT, astrophysics, cosmology.

In this research monopole-like solutions within special unitary group SU(2) Yang-Mills theory containing a doublet of nonlinear spinor fields will be presented. This will be of particular interest, so that the energy spectrum of such particle-like objects has a minimum. Such a minimum in the energy spectrum can be considered as a mass gap. Using the obtained results, one then may try to understand the nature of the mass gap in a more complicated situation in QCD. Thus, the mass gap Δ is the mass of the least massive particle predicted by the theory, so it is the energy gap between vacuum and first ground state. All these moments indicate **relevance of the problem** for the development of fundamental science, studied in this dissertation work.

The goals of the research: To obtain and investigate regular solutions of thick branes ($\mathcal{D}$– branes) in multidimensional space-time within the framework of $\mathcal{F}(R) = -\alpha R^n$ modified theories of gravity and study topologically trivial monopole-like solutions within the framework of SU(2) Yang-Mills theory containing a nonlinear doublet of spinor fields and demonstrate the presence of a minimum in energy spectrum (mass gap).

To achieve these goals, it is necessary to solve the following **objectives**:

– within the framework of $\mathcal{F}(R)$ modified theories of gravity obtain flat-symmetric solutions describing $\mathcal{D}$– branes in multidimensional space-time and investigate properties of such objects;

– obtain topologically trivial monopole-like solutions within the framework of SU(2) Yang-Mills theory containing a doublet of nonlinear spinor fields;

– study energy spectra of these solutions and show that they have a global minimum (mass gap).

Object of the research: modified theory of gravity, SU(2) Yang-Mills theory, nonlinear spinor fields and regular solutions in it.

Subject of research. Thick branes in $\mathcal{F}(\mathcal{R})$ modified theory and SU(2) Yang-Mills monopole with nonlinear spinor source.

Research methods. Numerical and analytical methods for studying nonlinear differential equations of modified theories of gravity, SU(2) Yang-Mills theory containing a doublet of nonlinear spinor fields.

Scientific novelty. The novelty and originality of research lies in the fact that**:**

– flat-symmetric solutions in modified theories of gravity, which describes thick branes with codim=1, having an Anti-De Sitter asymptotic are obtained;

– new Yang-Mills monopole-like objects with the source of nonlinear spinor, which feature is the presence a mass gap in the energy spectrum were obtained;

– it has been demonstrated that the main reason of the existence of such regular monopole-like solutions and a mass gap in such theory was the presence of a doublet of nonlinear spinor fields.

Defense Provisions:

1. In the theory of gravity with a modified Lagrangian $\mathcal{F}(R) = -\alpha R^n$ there are thick branes with anti-De Sitter asymptotics and with a special point located at the centre of the brane and existing at the following range of parameters of n: $1 < n < 2$.

2. SU(2) Yang-Mills theory with the source of doublet of nonlinear spinor field leads to the existence of topologically trivial monopole-like objects with $H \sim M/r^3$ asymptotic behavior of the SU(2) magnetic field.

3. Yang-Mills monopole with the source of nonlinear spinor field has a minimum in the energy spectrum (mass gap) – $(\widetilde{W}_t)_{min} = 5.812$ and 53.748 for the ground and first excited state for $\tilde{E} = 0.955$, the appearance of which is the consequence of nonlinearity of Dirac field.

Theoretical and practical significance of the research

The obtained results in this dissertation will contribute to a deeper understanding of the model of our Universe like a brane world. Moreover, the obtained regular solutions in gravitational theories are an interesting and necessary task for understanding the gravity interaction. Thick branes are hypothetical objects that may be discovered in the future. Therefore, the study of their properties is an important task in theoretical physics. As for the second part of the work, obtained new monopole-like solutions in SU(2) Yang-Mills theory aim to give a comprehensive account for understanding the properties of magnetic monopole. Magnetic monopoles are hypothetical particles that are actively researched. The investigation of their properties might shed light on the problem of symmetry of QED. Besides, these solutions can open up the door to investigate at the deeper level the concept of "mass gap", which is one of 7 Millennium Prize Problems.

Reliability and validity of the obtained results are confirmed by publications in journals of far abroad with high impact factors and in publications recommended by the Committee for Control in the Field of Education and Science of the Ministry

of Education and Science of the Republic of Kazakhstan, and in the proceedings of international scientific conferences of near and far abroad.

The personal contribution of the author lies in the fact that the entire volume of dissertation work, the choice of research method, problem solving and numerical calculations are performed by the author independently. Setting tasks and discussing the results were carried out jointly with scientific supervisors.

Approbation of the dissertation. The results obtained in the dissertation work were reported and discussed:

– at the second International Scientific and Practical Internet Conference "Actual issues of modern research" (2019, Nur-Sultan, Kazakhstan).

– at the International Scientific Conference of Students and Young Scientists "Farabi Alemi" (2020, Almaty, Kazakhstan);

– at 1st Electronic Conference on Universe (Online, 22-28 February 2021, China)

– in the Republican competition of research among universities of the Republic of Kazakhstan conducted by the Aktobe Regional University named after K. Zhubanov (1 place, 2021, Aktobe, Kazakhstan);

– and also discussed with Professor Jutta Kunz in the framework of international cooperation and internship (from November 2021 to February 2022, Oldenburg, Germany) .

Publications. Based on the materials of the dissertation, 8 printed works were published: 2 - publication in Kazakh journals, which are recommended by the Committee for Control in the Field of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan (KKSON MES RK) and 3 articles in journals of foreign countries with high impact factors included in the international information resource Web of Knowledge (Thomson Reuters, USA) and Scopus (Elsevier, the Netherlands); 3 works in the collections of International Scientific Conferences.

The volume and structure of the research. The thesis consists of an introduction, 4 sections, conclusion and list of references. The work is presented on 112 pages of printed text, contains 54 drawings and 2 tables. The list of references contains 161 items.

The main results of the dissertation work. Regular flat-symmetric solutions in multidimensional $f(R) = -\alpha R^n$ theory of gravity. From a physical point of view, these solutions present a model of our Universe as a thick brane with codim=1.

The properties of these branes depend on the following parameters: γ and δ, describing the properties of the solution in the center of the brane, and parameters α and n, describing a type of modified theory of gravity. To analyze the obtained solutions, phase portraits of the corresponding autonomous differential equations were constructed. The results show that brane solutions have AdS asymptotics. It

has been shown that the effective energy density T_0^0 is negative and its dependence on the parameter values $\gamma, \delta, \alpha, N$ was investigated.

As well regular finite-energy monopole-like solutions within SU(2) Yang-Mills theory containing the doublet of nonlinear spinor fields are obtained. The most important result of the investigation is that the energy spectrum possesses a global minimum, which can be interpreted as a mass gap, whose appearance is caused by the nonlinear spinor fields. The nonlinear Dirac equations possess regular solutions with finite energy and mass gap.

Additionally, typical behavior of the the eigenfunctions, the distributions of the color magnetic fields, total energy density $\tilde{\varepsilon}$ and energy spectrum of the system were analized. It was shown that asymptotic behavior of the radial magnetic field which is differs in principle from the Coulomb behavior of 't Hooft-Polyakov monopole. We wish to emphasize that the mass gap obtained in the present work can be considered as the QCD effect in non-QCD theory.